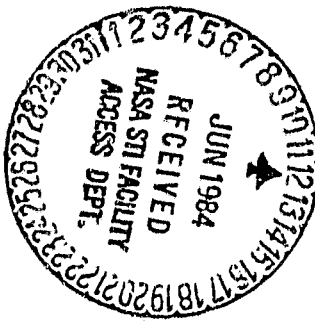


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Gerald R. North and Thomas J. Crowley

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National Aeronautics and
Space Administration

Goddard Space Flight Center
Greenbelt, Maryland 20771

MODELLING GLACIAL CLIMATES

Gerald R. North
Goddard Space Flight Center
National Aeronautics and Space Administration
Greenbelt, Maryland 20771

and

Thomas J. Crowley
Climate Dynamics Program
National Science Foundation
Washington, D. C. 20550

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MODELLING GLACIAL CLIMATES

by Gerald R. North and Thomas J. Crowley

ABSTRACT

Mathematical climate modelling has matured as a discipline to the point that it can be useful in paleoclimatology. As an example a new two dimensional energy balance model is described and applied to several problems of current interest. The model includes the seasonal cycle and the detailed land-sea geographical distribution. By examining the changes in the seasonal cycle when external perturbations are forced upon the climate system it is possible to construct hypotheses about the origin of midlatitude ice sheets and polar ice caps. In particular the model predicts a rather sudden potential for glaciation over large areas when the earth's orbital elements are only slightly altered. Similarly, the drift of continents or the change of atmospheric carbon dioxide over geological time can induce radical changes in continental ice cover. With the advance of computer technology and our improved understanding of the individual components of the climate system, it should be possible to test these ideas in far more realistic models in the near future.

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MODELLING GLACIAL CLIMATES

INTRODUCTION

The phenomenal progress made in paleoclimatology in the last century has been accomplished by combining ingenious proxy measurements with advanced techniques of paleontology, geochemistry, and mathematical statistics (e.g. Crowley, 1983). Although various investigators have employed simple climate models in their work over this period, most of these conceptual aids have been used to explain results rather than test scientific hypotheses. In this paper we shall argue that mathematical climate models are advancing to the stage where they will become active tools of the historical geologist in the next decade. This paper contains a brief tutorial description of one approach and shows a few examples of model solutions that exhibit dramatic behavior. We have liberally included our own view of the power and limitation of climate modelling and how we think the field may develop in the future.

We shall restrict our discussion to the climate system, which may be defined as the collection of all possible multivariate probability distributions relating to the atmosphere-ocean-cryosphere system. For practical purposes this can be thought of as the statistics (means, standard deviations, covariances, etc.) of such quantities as surface temperature, winds, the field of snow cover, the ocean currents, etc. The most straightforward approach to finding these quantities for given external governing factors is to

formulate the Newtonian mechanics problem for geophysical fluids and integrate the system of differential equations forward in time, later gathering the statistics. This strategy has proven plausible since the invention of the digital computer and the installation of a global network of observing stations for initializing the problem and for verification of the numerical solutions. The last thirty years of experience in building and experimenting with general circulation models of the global atmosphere have proven extremely useful in weather prediction. We now hope that these giant numerical models can also be used for the generation of climate experiments. In principle, we can make extremely long computer runs and collect the statistics as we would observational data. The output is a complete description of the present model climate as a control which can be used as a comparison against simulations of the climate under altered external forcing conditions.

Unfortunately, the problem proves to be far more difficult than this simple prescription suggests. The equations are so nonlinear and their linkages so complex that they are extremely unstable from a mathematical point of view. The result is that small errors amplify as the integration proceeds. The resulting degradation in effective determinism explains why meteorologists are not able to provide us with good weather forecasts two weeks in advance. It does appear, however, that many statistical quantities can be reliably computed from the models, and if we are willing

to relax our requirements to the estimation of these quantities we can develop a fruitful strategy for studying climate change. The statistics of the climate model may still not be a faithful analog of the actual climate. There are numerous uncertainties in the physics of many of the slower components such as oceanic current fluctuations and glacier dynamics. While these influences are negligible for weather forecasting, they are clearly of paramount importance for studies of long term climate change.

Given the above limitations, climate modellers have nonetheless attempted to apply their models to problems in paleoclimatology. Gates (1976) has performed experiments with a simplified two layer atmospheric model simulating the atmospheric circulation with boundary conditions taken from CLIMAP (1976) during the last glacial maximum (18,000 years ago). More recently, Manabe and coworkers have begun a series of experiments applicable to the Pleistocene glaciations (Manabe and Broccoli, 1984, and references therein). Kutzbach and Otto-Bliesner (1982) have attempted to simulate the monsoonal intensities for boundary conditions during the early Holocene (9000 years B.P.). Barron and Washington (1984) have applied general circulation models to the Cretaceous boundary conditions. While these efforts encourage us to pursue the modelling approach, large models are expensive to experiment with and they still are not reliable enough, especially in the polar regions, to consider them trustworthy. As an example of the state of the art, modelling groups working independently on the problem of

estimating the climatic impact of doubling the carbon dioxide concentration in the atmosphere obtain values for the global warming ranging from 1.5C to 4.0C (Schlesinger, 1983).

Considering the likelihood that the models are more like each other than they are like nature, we have indeed a very great uncertainty in climate sensitivity to external forcing.

Furthermore, the slower components of the system are just now being brought into the calculations, and it will be many years before these new features can be adequately formulated and tested.

In parallel with the development of the complex general circulation models there has arisen a wide range or hierarchy of intermediate level climate models yielding a wealth of information of varying degrees of relevance to paleoclimate. In some cases new effects were discovered first in the simplified models and later found to hold in more comprehensive schemes. Usually the approach has been to attempt to formulate equations for the climate statistics (e.g., ensemble means) directly instead of integrating forward in time and then gathering the statistics. For pilot studies and as guiding tools these simpler devices are likely to continue to play a major role in climate research over the next few years (cf. Schneider and Dickinson, 1974; or Saltzman, 1978).

All models incorporate a few adjustable coefficients. These parameters reflect our lack of understanding of the precise formulation of such diverse but important effects as

surface drag forces and variability in cloudiness. It is often possible to adjust these hidden parameters (fudge factors) in such a way that the solutions can be brought into agreement with most of the present day climatic observations even if parts of the model are incorrectly formulated. Thus, present day observations do not provide an adequate objective test of a model's validity. This looseness in model definition makes application of today's climate models to paleoclimate investigations extremely problematic, since the charge of ad hoc tuning of the model can be so easily leveled.

In view of the great uncertainties in climate model solutions and questions about the wisdom of generating theories for which there are no tests, it is legitimate to ask what, if any, are good candidate paleoclimate problems for climate modellers to pursue? In this paper we shall consider a few paleoclimate problems which we have examined in some detail in our own modelling activities and for which we think the effects are dramatic enough to warrant further consideration. The first question involves the onset of glaciation of the two present continental ice sheets--Greenland and Antarctica. Our model experiments suggest that ice sheets were triggered by the movement of the continental plates on time scales of millions of years. The second question pertains to the Pleistocene glaciations which have received so much recent attention because of the pivotal work of Hays, Imbrie, and Shackleton (1976). Direct statistical correlation methods were used to show that glaciations are

related to changes of the earth's orbital elements. In this case the physics of the forcing is so straightforward that it provides a rough calibration for model sensitivity. In both the Pleistocene and the earlier events other factors (e.g., carbon dioxide) are likely to be important but the radiation and geographical effects seem to be crucial. The underlying mechanism for glacial onset is the same for all the cases considered: a continuously variable forcing can produce a discontinuous transition of the climatic state (nonglacial to polar ice cap or polar ice cap to midlatitude glaciation). In order to understand this "critical point" behavior, it is first necessary to briefly describe the climate model. Then it is possible to describe the mechanism responsible for the discontinuous behavior. Finally the applications are discussed.

ENERGY BALANCE CLIMATE MODELS

The class of climate models we have chosen for illustration are the energy balance models. Since they have been discussed extensively in the literature (e.g., Schneider and Dickinson, 1974; North et al., 1981), a brief nonmathematical description will suffice here. As the name implies these models are an expression of the detailed conservation of energy for the earth-atmosphere system. The surface of the earth is divided into a grid of infinitesimal area segments each one of which represents an earth-atmosphere column. The conservation of energy for each segment is expressed as a relation among the fluxes of

insolation, terrestrial radiation to space, horizontal transport, and change in internal energy. This statement can be written as an equation, and if each term in the equation can be expressed as a functional of the surface temperature field, the field can be solved for. This solution is considered to be a simulation of the climate. The following few paragraphs describe the individual terms in the energy balance equation.

Heat energy can accumulate in an area segment by solar radiation absorption, which is the product of the local amount of sunlight incident at the top of the atmosphere and the absorptivity (co-albedo) of the local earth-atmosphere column. The sunlight depends upon season and location on the globe and is modulated by the earth's orbital properties. The absorptivity depends upon many factors, but we shall concentrate here on the strong contrast between ice and ice-free areas where the absorptivity can vary abruptly from about thirty to seventy percent.

On the real earth heat can enter or leave an area segment by horizontal transport due to winds or ocean currents. In our model heat transport is represented by a simple down gradient heat flux summed over the nearest neighbor area segments. Formally this term is the divergence of a gradient of temperature (diffusion), multiplied by a phenomenological coefficient which depends smoothly upon latitude. Such a dependence is a crude imitation of the latitude dependent features of the general circulation. Heat

is also lost from an area segment by the radiation of energy to space. This flux is usually modelled as a linear function of the surface temperature with empirical coefficients derived from satellite data.

The net gain of heat energy by a column of the earth-atmosphere system in a unit of time is the change in internal energy of the effective earth-atmosphere mass included. This change is usually taken as proportional to the change in surface temperature with a coefficient which is the effective heat capacity of the earth-atmosphere column. Over land at frequencies near that of the annual cycle and higher the solid surface absorbs very little heat and the relevant heat capacity is roughly that of the column of air. Over the open ocean heat is shared almost instantaneously with a wind driven mixed layer and consequently the relevant heat capacity for the earth-atmosphere system is that of the mass of a column of water about 75 meters deep, and in this situation that of the atmosphere (equivalent to about 2 meters of water) can be neglected. Where there is sea ice an intermediate value of effective heat capacity can be used to simulate the effects of leads, puddles, and the latent heat of fusion. These great contrasts in effective heat capacity (thermal inertia) of land, sea, and ice are responsible for modulating the intensity of the annual temperature cycle over the globe.

Readers interested in the quantitative formulation of the model and the method of solving it are referred to the paper by North et al. (1983). Also shown in that paper are

detailed illustrations of the model simulations of the present seasonal cycle and a comparison with the corresponding observations. To acquaint the reader with the faithfulness of the model in seasonal simulations we offer in Figure 1 the map of the Northern Hemisphere annual cycle amplitude for the model(a) and the observations(b). Since we have chosen a particularly favorable map to show (the mean annual temperature field does not agree as well with observations), we must remind the reader that this is an extremely simple model. Among other things we have ignored the many rich and fascinating dynamical properties of the atmospheric and oceanic motion fields and replaced them by what amounts to random short time scale wind fluctuations (diffusion). We also are ignoring or at best crudely imitating the detailed dynamical effects of hydrology as manifested in cloudiness changes, transport of latent heat, and most importantly for ice sheet studies the rate of precipitation in the form of snow. Even with these caveats in mind Figure 1 encourages us to use this model to speculate about the seasonal cycle in the past when the seasonal heating and the distribution of continental mass were different. But first it is necessary to explore the critical point properties of energy balance models.

CRITICAL POINTS AND SMALL ICE CAPS

The discontinuous response of climate models mentioned in the introduction results when more than one solution

obtains for the same external conditions on the system. Stated differently, for all external factors exactly the same there are two distinct stable climates possible. In particular, we shall see that often one solution is ice-free and another has a rather large stable ice cap. Under the right circumstances only a small change in the external parameters can carry the climate state to a point where only one of the distinct solutions is possible. The state may then have to undergo a transition to a qualitatively different regime to regain equilibrium. Reversing the forcing does not reverse the results. The climate remains in a qualitatively different regime until perturbed by a radically large change in forcing. For example, Figure 2 depicts the solution curve of a typical model in the neighborhood of a critical point by indicating the size of the ice cap versus some controlling parameter such as the solar constant. The cusp point (B) at the left most end of the ice-free portion of the curve is a critical point. In order to illustrate its effect upon climate change, suppose the initial state of the climate was at ice-free point A. Lowering the solar constant moves the equilibrium solution for the earth along the lower line to point B. The earth is still ice-free up to this stage. However, decreasing the solar constant by a small increment suddenly forces the equilibrium point to jump to the upper solution curve at point B'. In order to return to the ice-free state, the solar constant would have to be raised to another critical

point (C) after which the solution state must dramatically return to the ice-free branch.

In order to understand the critical point phenomenon we first consider a simpler model which has neither a seasonal cycle nor any longitudinal variation of temperature (i.e., a one dimensional model). These zonally symmetric mean annual models have been studied extensively in the last 15 years and their properties are well known (e.g., North et al., 1981). The main feature of interest in the present discussion is the ice-albedo feedback mechanism, which arises from the large albedo over ice-covered areas as opposed to ice-free areas. It is customary to choose a critical mean annual temperature (e.g., -10°C) below which ice is assumed to be locally present and above which it is not. In other words the absorptivity is a step function in temperature. Now imagine solving the model for the temperature as a function of latitude for sufficiently large values of the solar luminosity that no ice exists on the earth. That is to say, the temperature falls smoothly from the equator towards the poles and is above -10°C for all latitudes. This mathematical problem is particularly easy to solve since the absorptivity is constantly at its ice-free value and the energy balance equation is therefore linear. Figure 3a shows a typical curve of the ice-free temperature versus distance from the pole. Note that the temperature is about -8°C at its minimum at $r=0$ (rigorous mathematical details of this section can be found in the paper by North (1984)). This state corresponds to solution

point A of Figure 2.

Now we wish to ask about the solution if a small patch of ice is added at the pole. A disk of ice at the pole adds to the albedo of the earth-atmosphere system, and compared to the ice free state it effectively produces a steady sink of heat at the pole. It is possible to find the linear response of the climate model to this sink of heat. The result is a sizable thermal depression over a large distance from the sink, plotted schematically in Figure 3b for fixed (unit) area of the disk. The "influence function" is proportional to the area (strength) of the disk. For parameter values representative of the earth's transport and radiation the effective range (analogous to e-folding distance) of the influence function is about 20 degrees on a great circle. In other words a strong localized sink of heat will have a non negligible effect as much as 2000 kilometers away. Adding the curves in Figures 3a and 3b leads to the temperature curve in Figure 3c. By trial and error the strength (πa^2) was chosen such that the disk radius, a , also is the point where the perturbed temperature curve (3a) crosses the -10°C line. This latter assures that this disk size is an equilibrium solution. Mathematically this procedure is equivalent to finding the roots of an algebraic equation. It turns out that either one or three roots exist. To the left of B in Figure 2 only the large cap is possible. To the right of B (e.g., at A) there are three roots, one a large cap (A') and an intermediate ice cap (A'') which turns out to be unstable. States on portions of the curve with positive

slope are always unstable (Cahalan and North, 1979).

From the above analysis it is evident that the minimum size of a stable ice disk (corresponding to point C) is largely controlled by the characteristic horizontal range of the influence function. This minimum radius is typically of the order of 10 to 20 degrees on a great circle, depending upon the curvature of the temperature minimum. The analysis also enables us to offer a physical interpretation of the critical point phenomenon. As the solar luminosity is decreased the minimum of the curve in Figure 3a lowers and eventually becomes tangent to the horizontal critical line and after this there can no longer be an ice free solution. In this case only the finite sized cap just discussed will exist as an equilibrium solution to the equation. The merging of these two curves into tangency (equivalent to passing point B from right to left in Figure 2) is precisely the mechanism we intend to invoke for the initiation of large ice sheets when only small changes in the external conditions have occurred. Even if the change of an external parameter is small, it can have a large effect if the climatic state is near a critical point. Although the argument presented here is descriptive, it can be shown (North, 1984) that it rigorously holds in diffusive climate models with discontinuous albedo at the ice edge. Such discontinuous phenomena have been proposed in the past based upon empirical or intuitive arguments (e.g., Brooks, 1949; or Ives et al., 1975), but to our knowledge none have been discussed as

solutions to global mathematical climate models.

One further point needs to be emphasized: equilibrium models such as those discussed in this paper do not incorporate hydrology and therefore cannot predict the time required for the discontinuous transition to take place. Crossing a critical point for even a few years by a natural fluctuation will not ensure the complete transition. The model does not predict "instantaneous glacierization" since thousands of years may be required for the buildup of ice volume.

APPLICATIONS TO PAST CLIMATES

Now for application of the critical point phenomenon let us return to the seasonal model described earlier. Several authors including Köppen and Wegener (1924) and Milankovitch (1941) have stressed that it is the seasonal cycle of the temperature distribution rather than the annual mean that is important in the growth of continental ice sheets. We adopt the simple view that ice sheets in equilibrium cannot exist if summer temperatures go above freezing. This rather obvious oversimplification allows us to close the problem without having to consider the complication of moisture budgets. By considering the late summer as the key time for glacier formation, we are able to bring the seasonal cycle amplitude with its rich geographical features into play with the critical point phenomenon discussed in the last section. The analog of Figure 3a is now a contour map of the late

summer thermal field. Local minima of this field if just above 0C are candidates for critical points. Several factors can contribute to modifying the extremes of summer temperature and therefore the depth of the relative minima. Perhaps the most interesting involves the size and placement of land masses. In short, large land masses near the poles tend to have hot summers not supportive of ice growth; whereas, smaller land masses embedded in maritime regions will not get as hot in late summer and therefore may be favorable sites for ice sheet seeding.

The following applications are presented in their order of simplicity relative to the foregoing discussion. Let us begin by examining the temperature maps near the poles in the model simulations for ice free (low albedo) conditions at about one month past summer solstice (the warmest time of year). Figure 4 shows the January surface temperature map in the southern polar region. In this simulation no account is taken of the ice albedo of the continental land mass nor of its elevation above sea level (the observed temperatures near the poles are much colder because of these effects). In this case the model temperature field suggests that perennial snowcover (sea ice?) will persist in a very small patch off the coast of Antarctica at 180W. Our numerical solutions of the seasonal model show that allowing the snow to have its proper high albedo induces a transition to a large ice cap covering the entire continent and out to about latitude 70S. This model result suggests that only a large ice cap is to be

expected under present conditions (left of point B in Figure 2).

By contrast, tens of millions of years ago the earth was apparently much warmer, possibly because of higher atmospheric carbon dioxide concentrations (e.g., Berner et al., 1983). The model suggests that under the influence of a several fold increase in carbon dioxide an ice-free South Pole was possible as well as an ice cap climate. We speculate that the pole was at one time ice free and that as conditions changed, the climatic critical point was crossed resulting in the present Antarctic ice sheet. The necessary triggering parameter change could have been the separation of Australia from Antarctica about 40 million years ago. The separation leads to cooler summers in the continental interior because of the moderating influence of the oceans. Separating the continents also opens a new channel for circumpolar ocean currents and thereby reduces the heat transport from the tropical regions (Kennett, 1977). In either of these cases the separation is analogous to lowering the solar constant in Figure 2.

Our numerical model solutions therefore suggest that an ice free south pole is very nearly an equilibrium climate solution under present conditions, and that small changes in either atmospheric carbon dioxide or continental configuration could have induced the change to the present ice sheet. Model calculations (Figure 2) suggest that the process is not reversible, however, since an increase in carbon dioxide by a factor of about 10 would be necessary to

remove the ice sheet once it is in place (to advance from point B' to point C in Figure 2). Further details can be found in the paper by Crowley et al. (1984).

Now consider the Arctic region. Figure 5a shows the model July temperature simulation for present conditions and Figure 5b shows the corresponding map of observations. Note the minima in the Norwegian Sea and near the Bering Strait. These are due to the strong oceanic suppression of the seasonal cycle. In this case we encounter the first example of more than one relative minimum in the summer temperature field. Clearly an even richer climate solution structure is possible in this region. The deeper minimum in the Atlantic sector is an obvious candidate for explaining the Greenland ice sheet. It is important to realize that the model simulation does not take into account the effects of the warm Gulf Stream current. The depression in temperature is solely a result of the land-sea configuration. We suggest that the Greenland ice sheet is the result of the passage through a climatic critical point because of continental drift. It is not difficult to envision the deepening of the Atlantic minimum in Figure 5 through a critical point as the Norwegian Sea widened some 40 to 50 million years B. P. In fact, after rearranging the continents in the model to the configuration they had 50 million years ago (mid-Eocene) and examining the corresponding model solutions in late summer, we found that the near critical thermal minima no longer occurred over land masses such as present Greenland, and therefore there was

probably no large ice sheet in the Northern Hemisphere at that time. It is interesting that this conclusion results without reference to the carbon dioxide concentration in the atmosphere. Although the critical point mechanism gets rid of the ice sheet, it does not cause the global temperature to increase very much and it is likely that the carbon dioxide accessory will be necessary to account for large inferred global temperature increases 50 million years B.P. (e.g., Crowley, 1983).

In a previous study employing the model (North et al, 1983), it was shown that a discontinuous transition from the Greenland ice sheet to a much larger one occurred as the orbital elements were changed to their values 115,000 years B. P. The larger ice sheet was identified with the Laurentide ice sheet. It is now possible to explain that discontinuous transition with the critical point concept in conjunction with Figure 5. The weak relative minimum in the Pacific sector did not go critical when the Greenland ice sheet formed but is very near the critical point under present conditions. Natural orbital element variations are enough in the model to induce Laurentide-like transitions reminiscent of the Pleistocene data.

Before we argue that this result explains the Pleistocene glaciations, we must note that the Pacific sector minimum is less clearly identified in the observed summer temperature field (Figure 5b). This sobering fact causes us to continue to label this notion about the Pleistocene glaciations as

speculation. We suspect that this is not the precise mechanism involved but that some closely related phenomenon does occur; for a suggestion involving atmospheric dynamics see the paper by Crowley (1984). Clearly, the patchiness of the land in Northern Canada moderates the summers compared to those in Siberia and this is a factor in favoring North America in the ice sheet inception. It is surely no coincidence that the secondary minimum in the Arctic is so close to the critical point that even small changes such as the Milankovitch-Imbrie perturbations are sufficient to induce large equilibrium ice sheet solutions.

IMPLICATIONS

Questions relating to the inception of the great land based ice sheets afford a fertile problem area for research with climate models. The work presented here is at best preliminary, and further studies with both larger and smaller models are needed. In particular, we recommend testing these ideas in more comprehensive general circulation models where fewer idealizations and approximations are necessary. Such a test is not simple since the dynamical equations are especially poorly behaved near a critical point so that extremely long computer runs are necessary.

Along a parallel path there needs to be further study of the class of nonequilibrium problems where the accumulation and ablation of ice mass as well as its weight are explicitly taken into account. This problem is very difficult since even today's most comprehensive atmospheric

models cannot satisfactorily reproduce the precipitation regimes near the poles. However, steady progress is being made on this problem through continuous development of the general circulation models for application in other contemporary climate change problems. An additional hindrance to progress on the nonequilibrium problem is a lack of understanding of the physical processes outside the atmospheric component. For example, the formation and spread of oceanic deep water is only now being given a descriptive foundation, and climatic feedback mechanisms related to the adjustments of the earth's crust to glacial loading are poorly understood at present.

In defense of the equilibrium model approach taken in this paper we wish to remind the reader that although the detailed time evolution of climate cannot be predicted, fewer unknown parameters are necessary in the formulation. In particular, all tuning of the ice sheet accumulation and ablation rates can be dispensed with in equilibrium studies of this type. In fact, the few adjustable parameters in this model have been fixed throughout at the values needed to simulate the present seasonal cycle. In other words, we did not adjust the model solutions to agree with preconceived ideas.

Even if the effects suggested in this model study are born out in more comprehensive schemes, the sensitivity of the global temperature is still quite low. We have identified a mechanism for creating large ice sheets while

decreasing the global average temperature only about 1C. In other words, an ice free earth does not necessarily imply a particularly warm earth. If paleontological conclusions of much warmer climates are valid, other forcing mechanisms must be invoked. A large increase in atmospheric carbon dioxide is one obvious candidate.

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References:

- Barron, E. J. and W. M. Washington, 1984: The role of geographic variables in explaining paleoclimates: Results from Cretaceous climate model sensitivity studies. J. Geophys. Res., 89, 1267-1279.
- Berner, R. A., A. C. Lasaga, and R. M. Garrels, 1983: The carbonate-silicate geochemical cycle and its effect on atmospheric carbon dioxide over the past 100 million years. Am. J. Sci., 283, 641-683.
- Brooks, C. P., 1949: Climate Through the Ages. Dover, New York.
- CLIMAP, 1976: The surface of the ice-age earth. Science, 191, 1131-1137.
- Cahalan, R. and G. R. North, 1979: A stability theorem for energy-balance climate models. J. Atmos. Sci., 36, 1205-1216.
- Crowley, T. J., 1983: The geologic record of climatic change. Rev. Geophys. Sp. Phys., 21, 828-877.
- Crowley, T. J., 1984: Atmospheric circulation patterns during glacial inception: A possible candidate. Quat. Res., 21, 105-110.
- Crowley, T. J., G. R. North, D. A. Short, and J. G. Mengel, 1984: Climate model simulations of early Cenozoic glaciations. J. Geophys. Res. (submitted).
- Gates, W. L., 1976: Modeling the ice-age climate. Science, 191, 1138-1144.
- Hays, J. D., J. Imbrie, and N. J. Shackleton, Variations in the earth's orbit: Pacemaker of the ice ages, Science, 194, 1121-1132.
- Ives, J. D., J. T. Andrews, and R. S. Barry, 1975: Growth and decay of the Laurentide ice sheet and comparisons with Fennoscandia. Naturwiss., 62, 118-125.
- Kennett, J. P., 1977: Cenozoic evolution of Antarctic glaciation, the circum-Antarctic Ocean, and their impact on global paleoceanography. J. Geophys. Res., 82, 3843-3860.
- Koppen, W., and A. Wegener, 1924: Die Klimate der Geologischen Vorzeit, Gegrundet von Borntraeger, Berlin.

Kutzbach, J. E. and B. L. Otto-Bliesner, 1982: The sensitivity of the African-Asian monsoonal climate to orbital parameter changes for 9000 years B.P. in a low resolution general circulation model. J.Atmos.Sci., 39, 1177-1188.

Manabe, S. and A. J. Broccoli, 1984: The influence of continental ice sheets on the climate of an ice age. Annals of Glaciology, 5, (in press).

Milankovitch, M., 1941: Canon of insolation and the ice age problem (in Serbo-Croatian), Spec.Publ.132, R. Serbian Acad., Belgrade, Yugoslavia. (English translation, Israel Program for Sci. Translation, Jerusalem, 1969).

North, G. R., 1984: The small ice cap instability in diffusive climate models. J. Atmos.Sci. (submitted).

North, G. R., R. F. Cahalan, and J. A. Coakley, 1981: Energy balance climate models. Rev.Geophys.Sp.Phys., 19, 91-121.

North, G. R., J. G. Mengel and D. A. Short, 1983: Simple energy balance model resolving the seasons and the continents: Application to the astronomical theory of the ice ages. J.Geophys.Res., 88, 6576-6586.

Saltzman, B., 1978: A survey of statistical-dynamical models of the terrestrial climate. Adv. Geophys., 20, 183-303.

Schlesinger, M. E., 1983: A review of climate models and their simulation of CO₂-induced warming, Intern.J.Environ.Studies, 20, 103-114.

Schneider, S. H. and R. Dickinson, 1974: Climate modelling. Rev. Geophys. Sp. Phys., 447-493.

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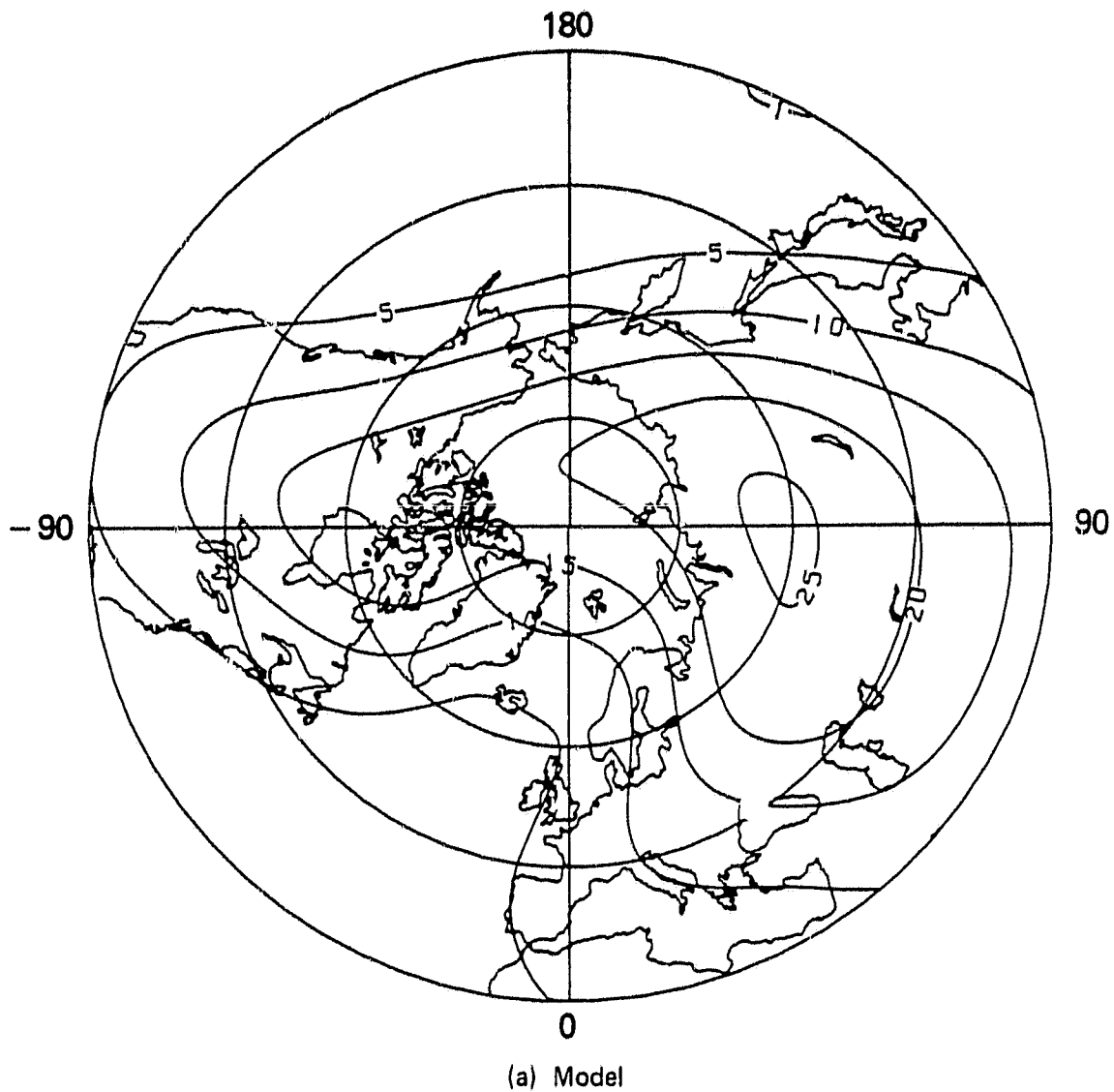
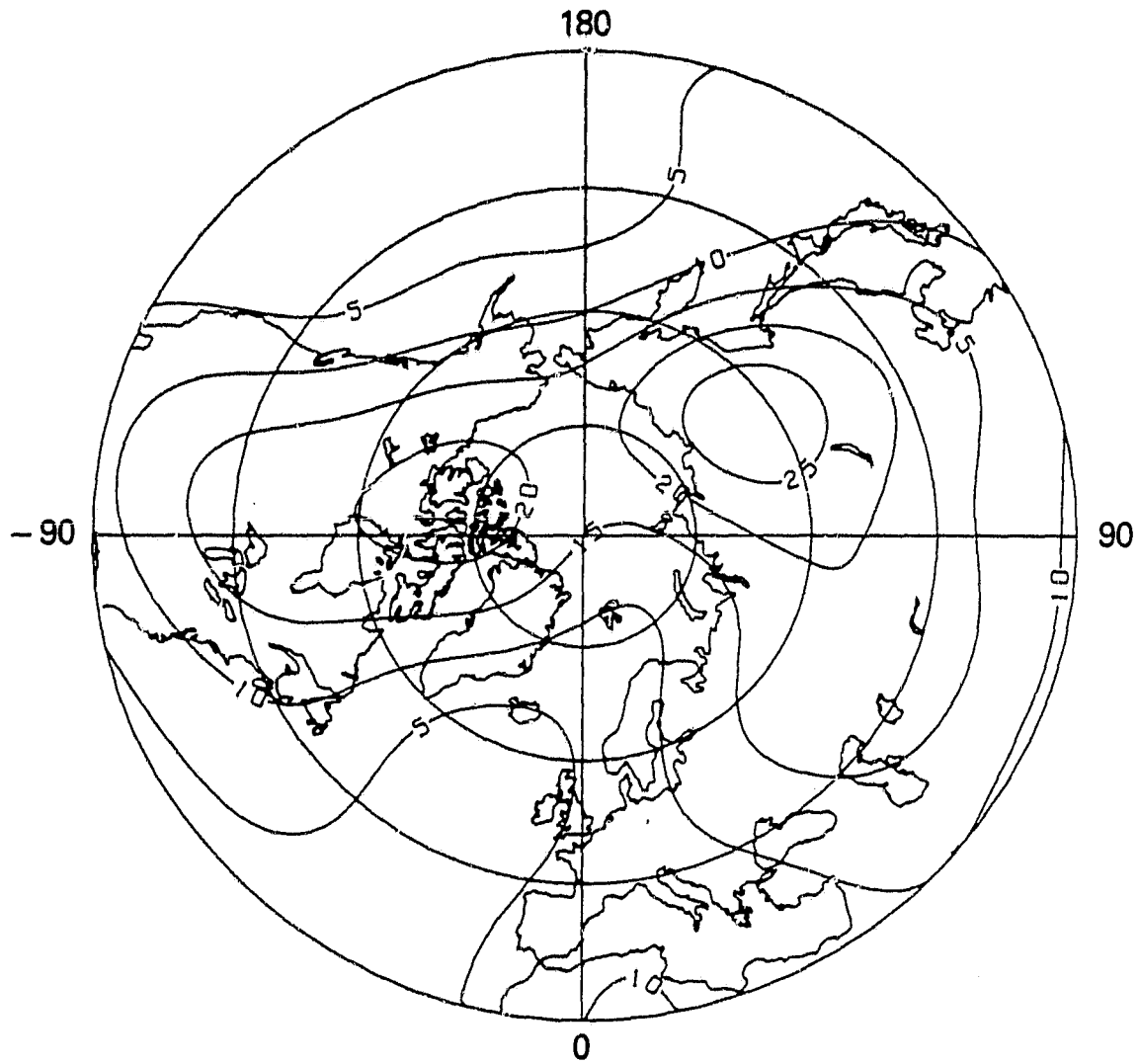


Figure 1a. The amplitude of the seasonal cycle (annual harmonic only) of surface temperature.

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(b) Observations

Figure 1b. The amplitude of the seasonal cycle (annual harmonic only) of surface temperature.

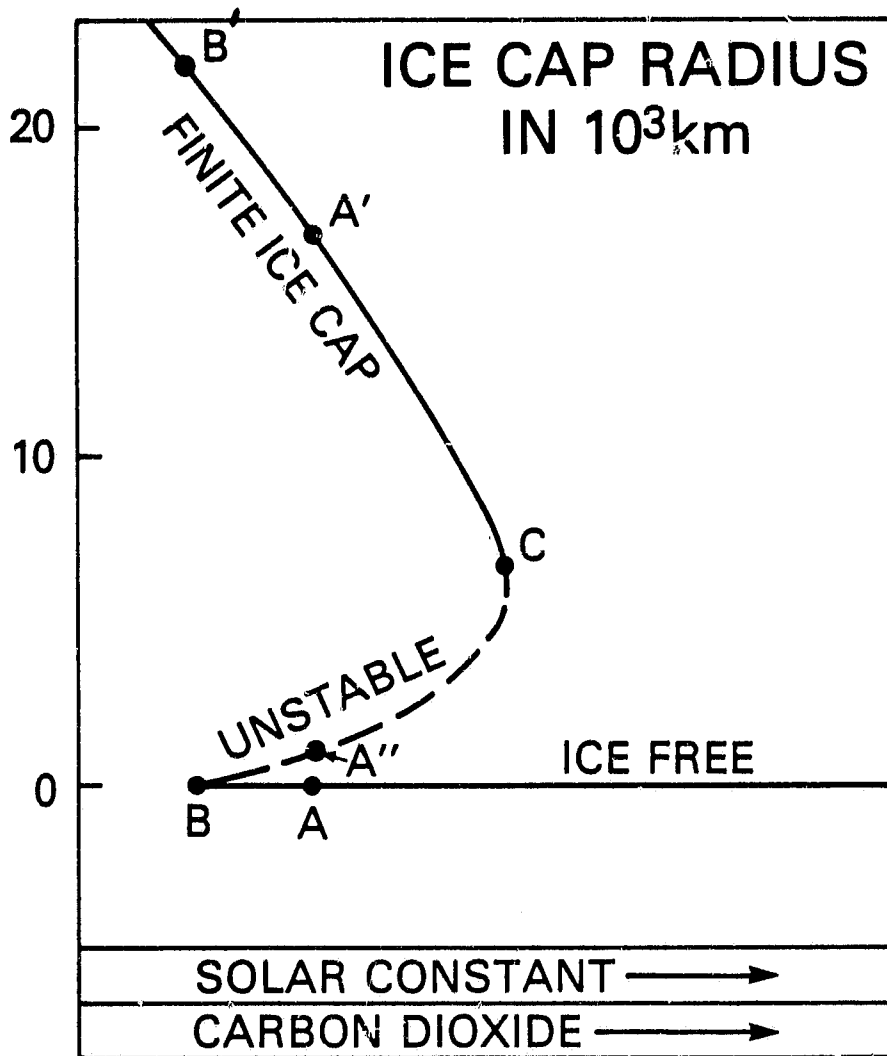


Figure 2. Schematic graph of equilibrium solutions of ice feedback models. The dependent variable is ice cap radius and the independent variable is the solar constant increasing to the right. Another radiative parameter having essentially the same effect is the carbon dioxide concentration which may also be taken as increasing to the right.

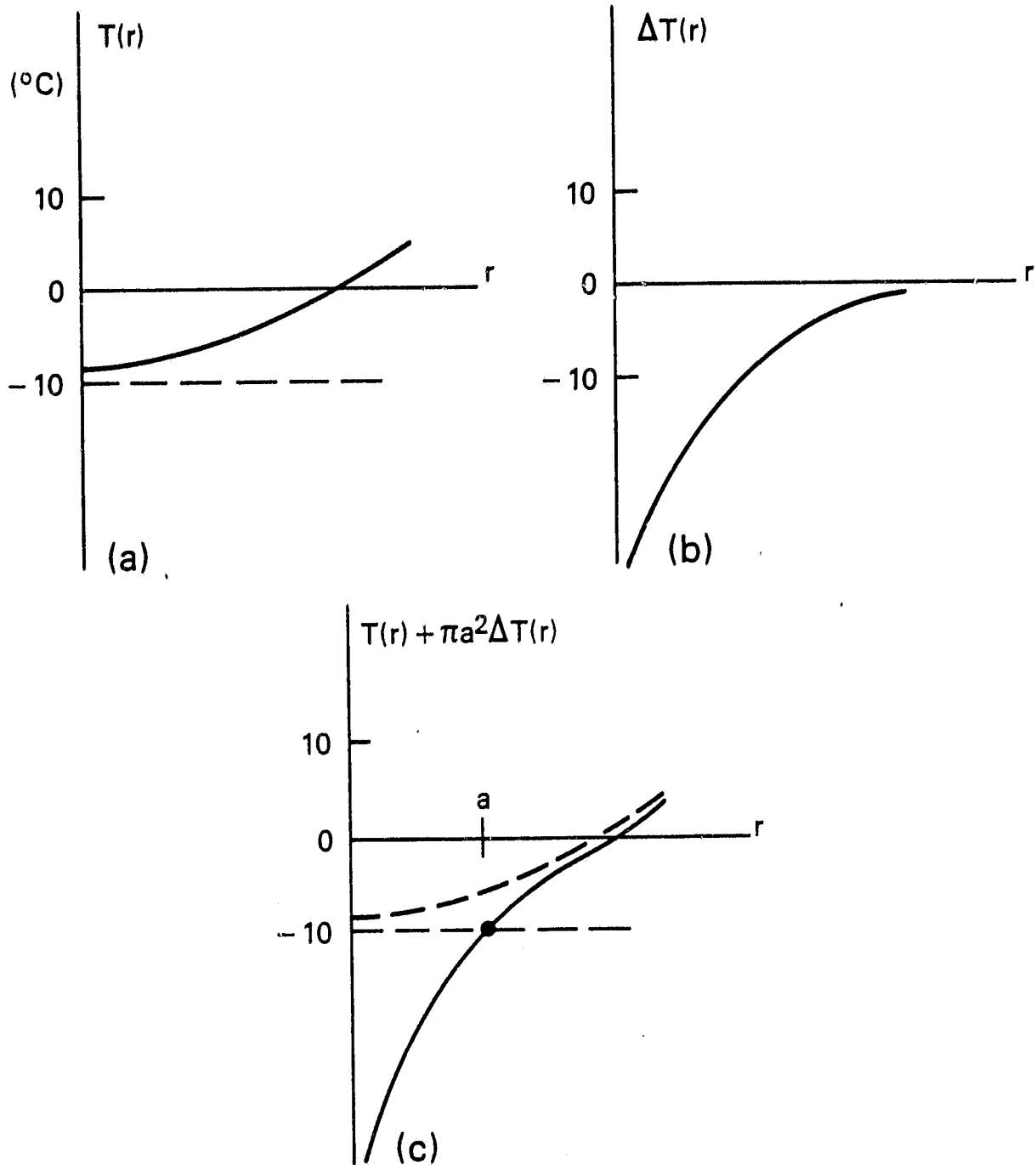


Figure 3. Temperature solutions versus distance from the pole. (a) The temperature for a climate state corresponding to point A in Figure 2. (b) The depression of temperature due to a point sink of heat at the pole corresponding to a small patch of ice. The depression is everywhere approximately proportional to the ice disk area. (c) The sum of curves in (a) and (b) where the size of the ice disk has been chosen to be of radius a and this value also is the correct equilibrium value for an ice cap corresponding to point B in Figure 2.

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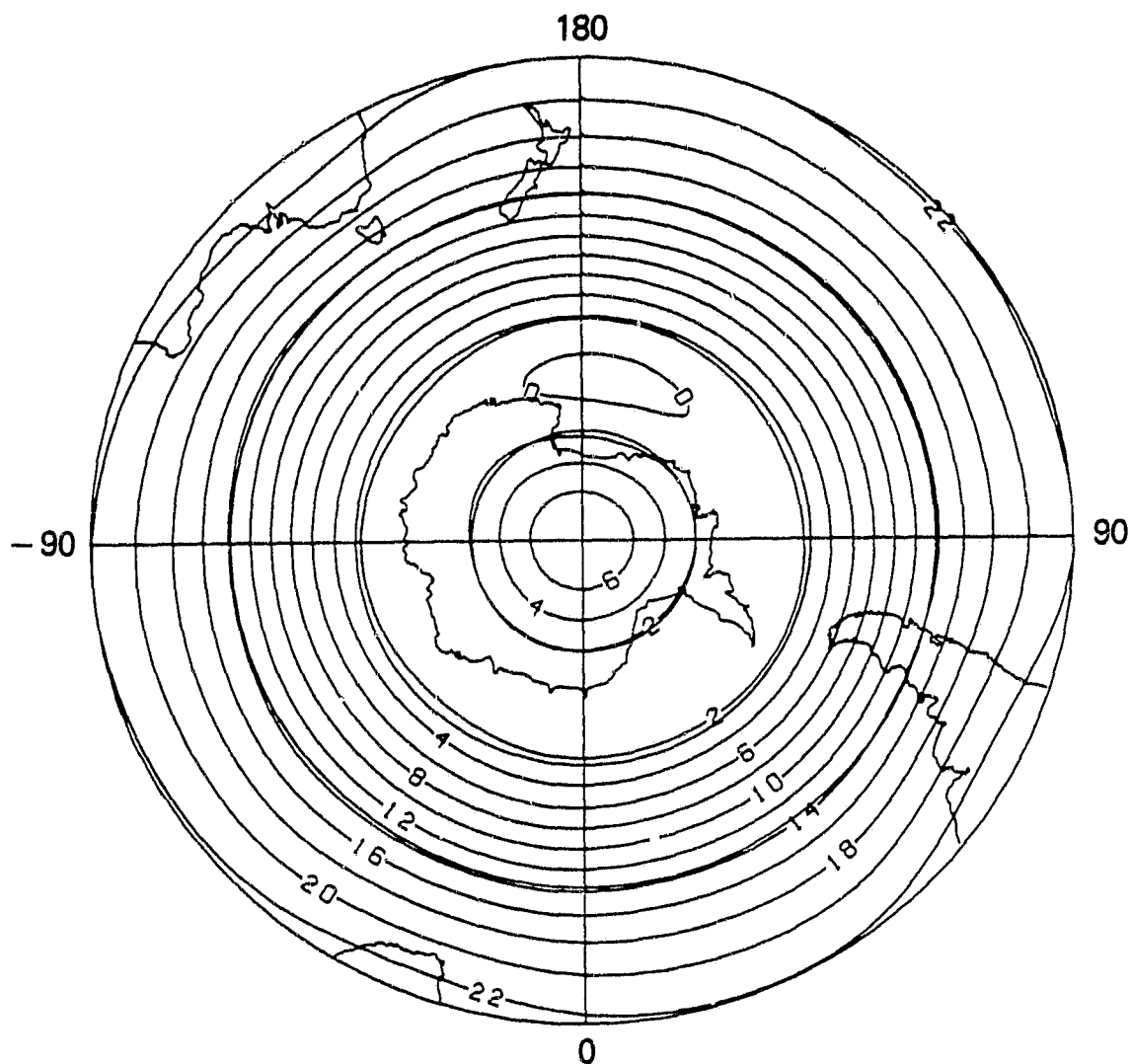


Figure 4. Contour map of the model simulated January temperature for the Southern Polar region for ice free albedo and no elevation of the continent.

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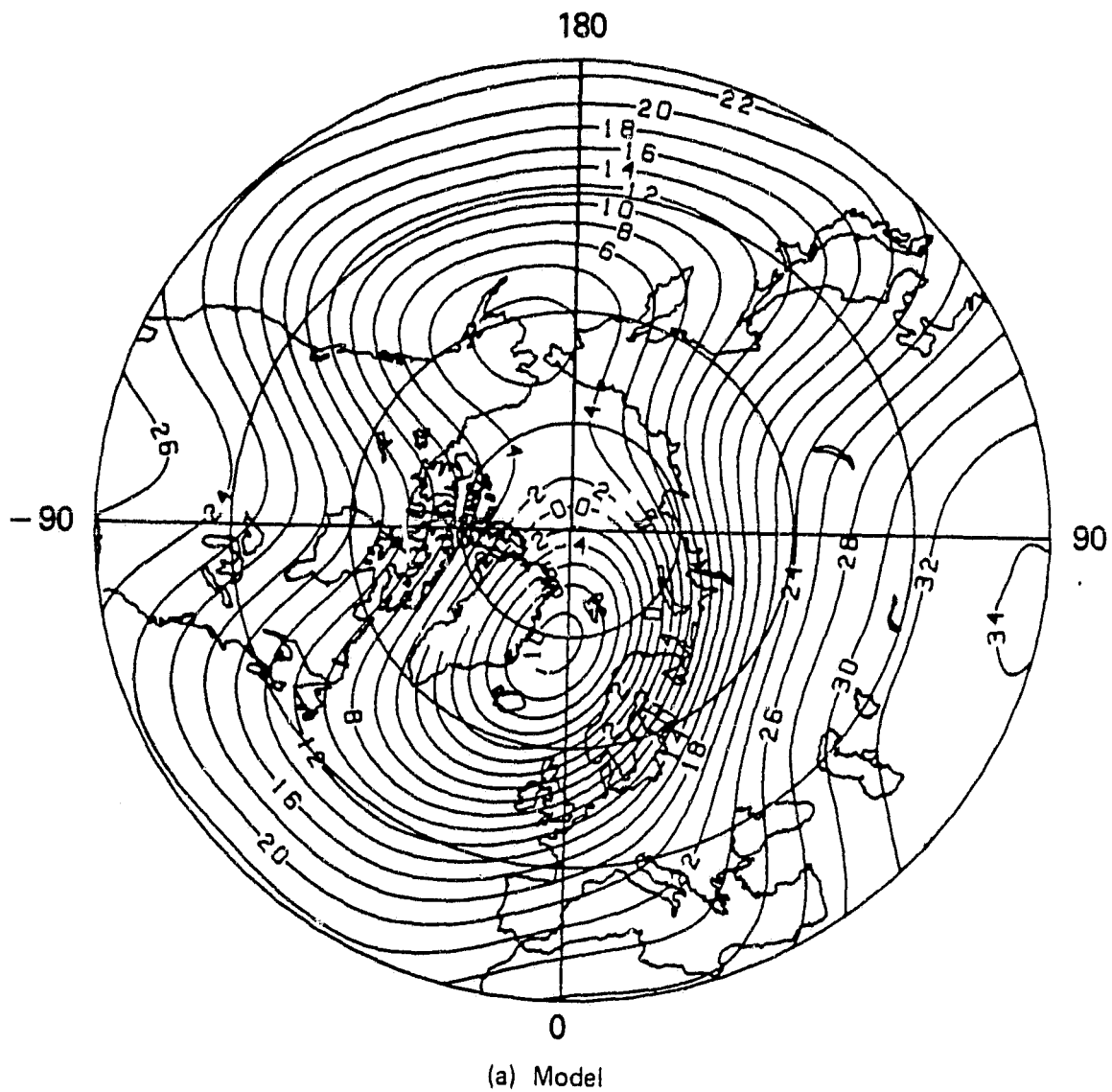
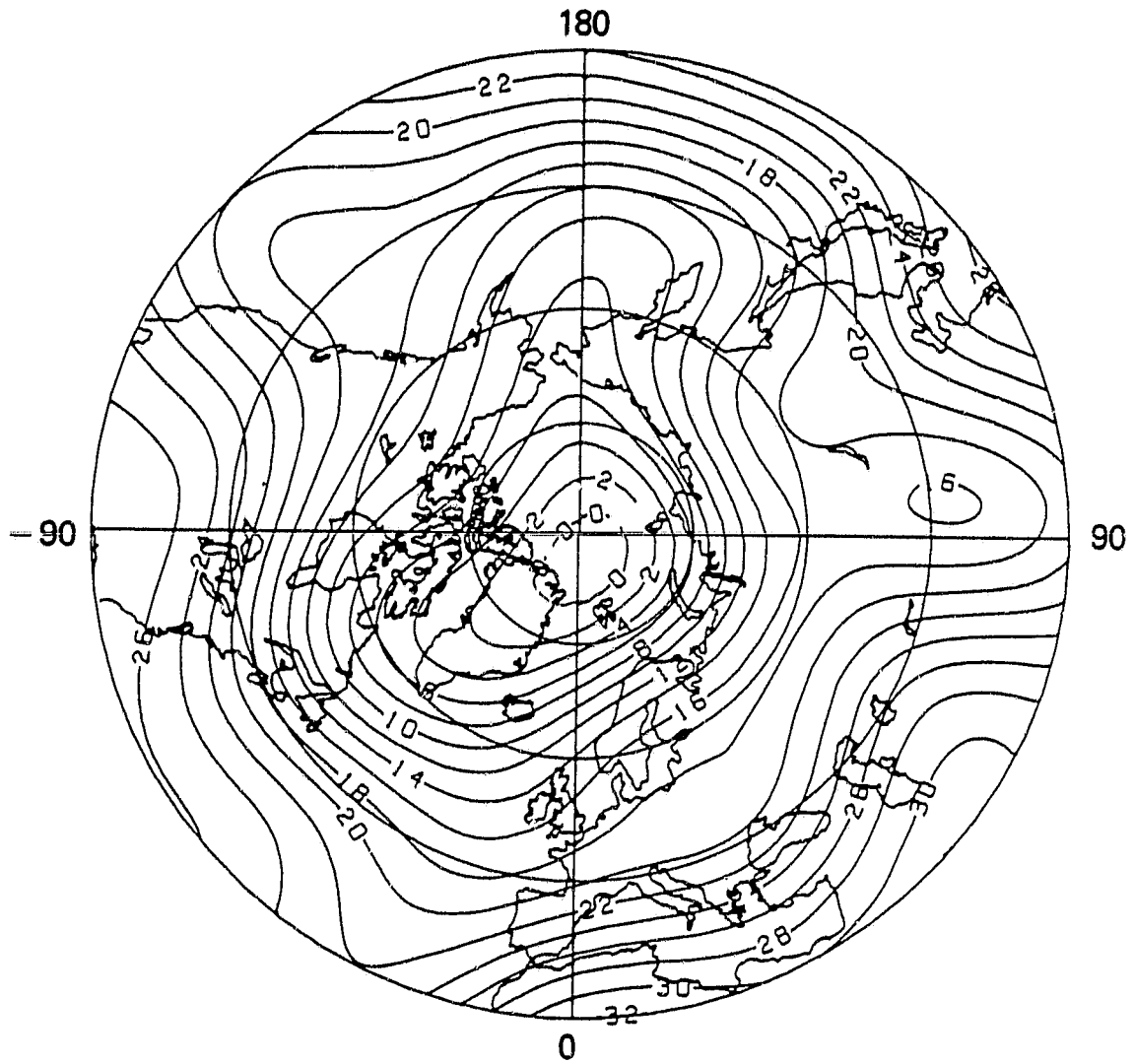


Figure 5a. The July temperature distribution for present conditions.



(b) Observations

Figure 5b. The July temperature distribution for present conditions.

FIGURE CAPTIONS

Figure 1. The amplitude of the seasonal cycle (annual harmonic only) of surface temperature. (a) model. (b) observations.

Figure 2. Schematic graph of equilibrium solutions of ice feedback models. The dependent variable is ice cap radius and the independent variable is the solar constant increasing to the right. Another radiative parameter having essentially the same effect is the carbon dioxide concentration which may also be taken as increasing to the right.

Figure 3. Temperature solutions versus distance from the pole. (a) The temperature for a climate state corresponding to point A in Figure 2. (b) The depression of temperature due to a point sink of heat at the pole corresponding to a small patch of ice. The depression is everywhere approximately proportional to the ice disk area. (c) The sum of curves in (a) and (b) where the size of the ice disk has been chosen to be of radius a and this value also is the correct equilibrium value for an ice cap corresponding to point B in Figure 2.

Figure 4. Contour map of the model simulated January temperature for the Southern Polar region for ice free albedo and no elevation of the continent.

Figure 5. The July temperature distribution for present conditions. (a) model. (b) observations.